

Math 262
Take home final

Due: 2pm, Thursday, December 9, 2004

1. **(a)** State your favorite version of the regularity lemma.
(b) What is your favorite application of the regularity lemma? Why?

2. For given real numbers $0 < \alpha < 1/\beta < 1$, we want to find k as small as possible so that for any n large enough, there exists a k -regular bipartite graph with vertex set $A \cup B$, $|A| = |B| = n$, satisfying the property that every subset $X \subseteq A$ with $|X| \leq \alpha n$ has at least $\beta|X|$ neighbors in B .
(a) For the existence proof, use the probabilistic methods (in this case, essentially a counting argument). What is the smallest possible k for which such a bipartite graph exists?
(b) For the explicit construction, use spectral methods. What is the smallest possible k for constructing such a bipartite graph?